

CH: 4 OSCILLATIONS

PAGE NO.	
DATE	

①

1) Restoring force:

$$F = -kx \Rightarrow F = -k(-mx\omega^2) = -m\omega^2 x$$

2) Differential equation of linear S.H.M.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

3) Displacement in S.H.M.

$$x = A \sin(\omega t + \alpha)$$

a) Mean position: $\alpha = 0 \Rightarrow x = A \sin \omega t$

b) Extreme position: $\alpha = \pi/2 \Rightarrow x = A \cos \omega t$

4) Velocity in S.H.M.: $v = \pm \omega \sqrt{A^2 - x^2}$

Acceleration in S.H.M.: $a = -\omega^2 x$

5) Period in S.H.M.:

$$T = 2\pi \sqrt{\frac{m}{k}} = \frac{2\pi}{\omega}$$

6) Energy in S.H.M.:

$$a) P.E. = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2$$

$$b) K.E. = \frac{1}{2} k(A^2 - x^2) = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$c) T.E. = K.E. + P.E. = \frac{1}{2} kA^2 = \frac{1}{2} m\omega^2 A^2$$

7) Composition of S.H.M. and simple motion

a) Resultant equation:

$$x = R \sin(\omega t + \delta)$$

b) Resultant amplitude:

$$R = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos(\alpha_1 - \alpha_2)}$$

c) Phase: $\delta = \tan^{-1} \left(\frac{a_1 \sin \alpha_1 + a_2 \sin \alpha_2}{a_1 \cos \alpha_1 + a_2 \cos \alpha_2} \right)$

8) Period of simple pendulum:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

(2)

9) Frequency of S.H.M.:

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{\omega}{2\pi}$$

10) Maximum Velocity $\Rightarrow V_{max} = A\omega$ Maximum Acceleration $\Rightarrow a_{max} = A\omega^2$ 11) Force constant: (Restoring force per unit displacement): $\Rightarrow K = m\omega^2$ 12) Displacement of particle performing S.H.M. (projection along diameter = $2a$):

$$x = a \cdot \cos(\omega t + \phi)$$

13) For an oscillating massless spring: $T = 2\pi \sqrt{\frac{m}{K}}$ 14) When two vertical springs of force constants K_1 & K_2 are joined -

a) in series: $K = \frac{K_1 K_2}{K_1 + K_2}$

b) in parallel: $K = K_1 + K_2$

15) If spring is cut in 'n' equal parts then force constant of one part: $K' = nK$ 16) When bob of simple pendulum is immersed in liquid of density σ : $T = 2\pi \sqrt{\frac{l}{g'}}$ where $g' = g(1 - \frac{\sigma}{\rho})$ where $\rho \Rightarrow$ Density of bob.① In the differential equation $\frac{d^2x}{dt^2} + \omega^2 x = 0$,for a simple harmonic motion, the term ω^2 represents -

- a) restoring force per unit mass
- b) restoring force per unit displacement
- c) restoring force per unit mass per unit displacement.

d) acceleration per unit mass per unit displacement.

\Rightarrow (c) $\omega^2 \Rightarrow$ restoring force per unit mass per unit displacement. ($\because \omega^2 = \frac{F}{m}$)

(4)

② The physical quantity whose SI unit is the same as that of the force constant is —

- (a) pressure
- (b) surface tension
- (c) potential energy
- (d) torque

→ (b) Force constant $\frac{\text{restoring force}}{\text{per unit displacement}}$

$$K = \frac{F}{x} = \frac{N}{m}$$

$$K = m\omega^2 = m \frac{v^2}{r^2}$$

$$= \frac{\text{kg} \cdot \text{m}^2/\text{s}^2}{\text{m}^2}$$

$$\text{Surface tension, } T = \frac{F}{L} = \frac{N}{m}$$

$$T = \frac{F}{L} = \frac{m \cdot a}{L}$$

$$= \frac{\text{kg} \cdot \text{m} \cdot \text{s}^2}{\text{m} \cdot \text{s}^2}$$

③ The equation of displacement of a harmonic oscillator is $x = 3 \sin \omega t + 4 \cos \omega t$. The amplitude of the particle will be —

- (a) 2
- (b) 5
- (c) 7
- (d) 12

→ (b) $x = 3 \sin \omega t + 4 \cos \omega t$

$$= 3 \sin \omega t + 4 \cos \left(\frac{\pi}{2} + \omega t \right)$$

$$\text{Now, } a_1 = 3, a_2 = 4, \alpha_1 - \alpha_2 = \frac{\pi}{2}$$

∴ Resultant amplitude is $R = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos(\alpha_1 - \alpha_2)}$

$$(Q+P) R = \sqrt{3^2 + 4^2 + 2(3)(4) \cdot \cos \frac{\pi}{2}}$$

$$= \sqrt{9 + 16 + 0} = \sqrt{25} = 5$$

④ Two simple harmonic motions are represented as $y_1 = 10 \sin \omega t$ & $y_2 = 5 \sin(\omega t + \pi/2)$. The ratio of the amplitudes of y_1 & y_2 is —

- (a) 1:1
- (b) 1: $\sqrt{2}$
- (c) $\sqrt{2}$:1
- (d) 1:2

→ (c) $y_1 = 10 \sin \omega t \therefore a_1 = 10$

$$y_2 = S \sin \omega t + S \cos \omega t$$

$$= S \sin \omega t + S \sin\left(\frac{\pi}{2} + \omega t\right)$$

$$\therefore \alpha_2 - \alpha_1 = \frac{\pi}{2} \quad \therefore \alpha_2 = \sqrt{S^2 + S^2 + \cos^2 \frac{\pi}{2}} = \sqrt{2S^2} = \sqrt{50}$$

$$= 5\sqrt{2}$$

$$\therefore \frac{\alpha_1}{\alpha_2} = \frac{10}{5\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\therefore \alpha_1 : \alpha_2 = \sqrt{2} : 1$$

- ⑤ When a particle performs a U.C.M. of diameter 10 cm, its projection along the diameter of the circle, performs a S.H.M. of amplitude _____

a) 10 cm b) 5 cm c) 20 cm d) 2.5 cm

\Rightarrow (b) Particle performs U.C.M. at instants

\rightarrow ad min (Diameter $\equiv 2a = 10$ cm)

∴ (b) Projection along diameter, amplitude $= a = \frac{10}{2} = 5$ cm.

- ⑥ Which of the following expressions does not represent a simple harmonic motion?

a) $x = A \tan(\omega t + \phi)$ b) $x = A \sin(\omega t + \phi)$
 c) $x = A \sin \omega t \cdot \cos \omega t + \phi$ d) $x = B \cos(\omega t + \phi)$

\Rightarrow (a) sin, cos & combination of sin & cos represents a S.H.M.

$\therefore x = A \tan(\omega t + \phi)$ doesn't represent

S.H.M. amplitude increased slowly out

- ⑦ A particle executes a linear S.H.M. of amplitude 8 cm & period 2 s. Then magnitude of its maximum velocity is _____

a) 8π cm/s b) 16π cm/s c) 4π cm/s d) 2π cm/s

→ (a) Maximum velocity, $v_{max} = A\omega$

$$= Ax \frac{2\pi}{T}$$

$$= 8 \times \frac{2\pi}{2} = 8\pi \text{ cm/s}$$

⑧ The displacement of a particle performing a S.H.M. is given by

$$x = 0.5 \sin 100\pi(t + 0.05)$$

where x is in metres & t is in second. Its periodic time in second is —

- a) 0.01 s b) 0.02 s c) 0.1 s d) 0.5 s

→ (b) $x = 0.5 \sin 100\pi(t + 0.05)$

$$= 0.5 \sin(100\pi t + 5\pi)$$

Comparing with $x = A \sin(\omega t + \alpha)$, we get

$$\omega = 100\pi \text{ rad/sec}$$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{1}{50} = 0.02 \text{ sec.}$$

⑨ The maximum velocity & maximum acceleration of a body moving in a simple harmonic manner are 2 m/s & 4 m/s². The angular velocity of the body is —

- a) 4 rad/s b) 3 rad/s c) 2 rad/s d) 1 rad/s

→ (c) In S.H.M., $v_{max} = A\omega$

$$v_{max} = A\omega^2$$

$$\therefore \omega = \frac{A\omega^2}{A} = \frac{4}{2} = 2 \text{ rad/s}$$

⑩ The amplitude & the time period in a S.H.M. is 0.5 cm & 0.4 s respectively. If the initial phase is $\frac{\pi}{4}$ radian, then the equation of motion of SHM is given by —

a) $y = 0.5 \sin(5\pi t)$

b) $y = 0.5 \cos(5\pi t)$

c) $y = 0.5 \sin(2\pi ft)$ d) $y = 0.5 \cos(2\pi ft)$

\Rightarrow (b) $a = 0.5 \text{ cm}, T = 0.4 \text{ s}, \omega = \pi/2 \text{ rad}$

\therefore Equation of motion of SHM is

$$y = a \cdot \sin(\omega t + \alpha)$$

$$= a \cdot \sin\left(\frac{2\pi t}{T} + \alpha\right)$$

$$= 0.5 \cdot \sin\left(\frac{2\pi t}{0.4} + \frac{\pi}{2}\right)$$

$$= 0.5 \sin(5\pi t + \pi/2)$$

$$= 0.5 \cos(5\pi t) \quad \left\{ \because \sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta \right.$$

- (11) The acceleration of a particle performing a linear S.H.M. is 16 cm/s^2 , when it is at a distance of 4 cm from the mean position. The period of S.H.M. is —

a) 6.28 sec b) 1.57 sec c) 5 sec d) 3.14 sec

\Rightarrow (d) Acceleration (Magnitude) in S.H.M. is

$$a = \omega^2 x \quad \left\{ \begin{array}{l} a = 16 \text{ cm/s}^2 \\ x = 4 \text{ cm} \end{array} \right.$$

$$\therefore \omega^2 = 4$$

$$\therefore \omega = 2 \text{ rad/s}$$

$$\text{Now, } T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \text{ sec} = 3.14 \text{ sec}$$

- (12) The velocity of a particle executing a S.H.M. of frequency 10 oscillations/sec is $11\pi \text{ m/s}$, when it is at the mean position. Up to what maximum distance, the particle will be displaced?

a) 2 cm b) 3 cm c) 4 cm d) 5 cm

\Rightarrow (d) $n = 10 \text{ osci/sec}, v = \pi \text{ m/s}$, Maxm distance upto which particle displaces

Particle is at mean position, $\therefore x=0$

$$\text{Now, } \dot{x}v = \omega \sqrt{A^2 - x^2} \text{ and } \ddot{x} = -\omega^2 x$$

$$\text{At mean pos. } \therefore v = \omega A \quad (\because x=0)$$

$$\text{Again, } \dot{x}v = 2\pi n A$$

$$\therefore A = \frac{v}{2\pi n} = \frac{\omega}{2\pi \times 10} = \frac{1}{20} \text{ m} = 0.05 \text{ m}$$

$$= 5 \text{ cm}$$

- (13) The frequency of a linear SHM-oscillator is to be doubled. For this the mass should be _____.
- doubled
 - halved
 - reduced to one fourth of its original value
 - increased to four times.

$$\Rightarrow (c) T = 2\pi \sqrt{\frac{m}{K}} \quad \therefore n = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$$

$$\therefore n \propto \frac{1}{\sqrt{m}}$$

$$\frac{n_2}{n_1} = \sqrt{\frac{m_1}{m_2}}$$

$$\therefore \frac{2n_1}{n_1} = \sqrt{\frac{m_1}{m_2}} \quad \text{To (Given)} \quad (\because n_2 = 2n_1)$$

$$\therefore 2 = \sqrt{\frac{m_1}{m_2}} \quad \text{squaring, } \frac{m_1}{m_2} = 4$$

$\therefore m_2 = \frac{m_1}{4}$ Reduced to $\frac{1}{4}$ of original.

- (14) A body of mass 980 gram is made to oscillate on a spring of force constant 4.9 N/m, then the angular frequency of the body is —
- $\sqrt{2}$ rad/s
 - $\sqrt{3}$ rad/s
 - $\sqrt{5}$ rad/s
 - $\sqrt{7}$ rad/s

$$\Rightarrow (c) m = 980 \text{ g} = 0.98 \text{ kg}, K = 4.9 \text{ N/m}$$

$$K = m\omega^2 \text{ or } \omega^2 = \frac{K}{m}$$

$$\therefore \omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{4.9}{0.98}} = \sqrt{\frac{498}{98}} = \sqrt{5} \text{ rad/s}$$

(15) The instantaneous displacement of a simple harmonic oscillator is given by $x = A \cos(\omega t + \frac{\pi}{4})$. At what time, its speed will be maximum?

- a) $\frac{\pi}{2\omega}$ b) $\frac{\pi}{4\omega}$ c) $\frac{2\pi}{\omega}$ d) $\frac{\pi}{\omega}$

$$\Rightarrow (b) \quad x = A \cos(\omega t + \frac{\pi}{4})$$

Velocity, $v = \frac{dx}{dt} = -A\omega \sin(\omega t + \frac{\pi}{4})$

The speed will be maximum, when

$$\sin(\omega t + \frac{\pi}{4}) = 1$$

$$\text{i.e. } \omega t + \frac{\pi}{4} = \frac{\pi}{2}$$

$$\therefore \omega t = \frac{\pi}{2} - \frac{\pi}{4} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$\therefore t = \frac{\pi}{4\omega}$$

(16) The maximum velocity for a particle in S.H.M. is 0.16 m/s & the maximum acceleration is 0.64 m/s². The amplitude of S.H.M. is

- a) 4×10^{-2} m b) 4×10^1 m
 c) 4×10^0 m d) 4×10^{-3} m

$$\Rightarrow (a) \quad \frac{v_{\max}}{a_{\max}} = \frac{\omega^2 A}{\omega A} = \frac{0.64}{0.16} = 4$$

at above eq. since $\omega = 4$ rad/s

Now, $v_{\max} = A\omega$ $\therefore A = \frac{v_{\max}}{\omega}$

$$\therefore A = \frac{0.16}{4} = 0.04 \text{ m}$$

but $A = 4 \times 10^{-2}$ m

(17) A particle is executing a linear S.H.M & its differential equation is $\frac{d^2x}{dt^2} + \alpha x = 0$. Its time period of motion is

$$a) \frac{2\pi}{\omega}$$

$$b) 2\pi\omega$$

$$c) 2\pi\sqrt{\omega}$$

$$d) \frac{2\pi}{\sqrt{\omega}}$$

$$\Rightarrow (d) \frac{d^2x}{dt^2} + \omega^2 x = 0$$

Comparing this with $\frac{d^2x}{dt^2} + \omega^2 x = 0$, we get

$$\omega^2 = \omega \quad \therefore \omega = \sqrt{\omega}$$

$$\text{Now, } T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega}}$$

- (18) A particle is performing a linear S.H.M. along x-axis with amplitude 4 cm & time period 1.2 s. The minimum time taken by the particle to move from x = +2 cm to x = +4 cm & back again is given by -
- a) 0.6 s b) 0.4 s c) 0.3 s d) 0.2 s

$$\Rightarrow (b)$$

$$A = 4 \text{ cm}, T = 1.2 \text{ s}$$

(a.i) Time for O-B, B-O, O-C & C-O is T.

(b.i) Time to go from O to B = $\frac{T}{4}$

$$\text{Time to go from O to A, } \frac{T}{4} = 0.3 \text{ s}$$

Time to go from A to B, period

$$x = A \sin(\omega t) = 0$$

$$\text{i.e. } x = A \sin\left(\frac{2\pi t}{T}\right)$$

$$\therefore 0 = 4 \sin\left(\frac{2\pi t}{T}\right)$$

$$\text{minimum period} = \sin\left(\frac{2\pi t}{T}\right)$$

$$\therefore \sin\frac{\pi}{6} = \sin\frac{2\pi t}{T} \quad \therefore \frac{\pi}{6} = \frac{2\pi t}{T}$$

$$\therefore t = \frac{\pi T}{12\pi} = \frac{T}{12}$$

$$\text{Initial time taken} = 1.2 = 0.1 \text{ s}$$

$$\therefore \text{Time to go from A to B} = 0.3 - 0.1 = 0.2 \text{ s}$$

$$\therefore \text{Required time} = 0.2 + 0.2$$

$$= \cancel{0.4} \quad 0.4 \text{ s}$$

- (19) A point mass oscillates along the x-axis according to the law,
 $x = x_0 \cos(\omega t - \pi/4)$.

If the acceleration of the particle is written as -

$$a = A \cos(\omega t + \delta), \text{ then}$$

- a) $A = x_0$, $\delta = -\frac{\pi}{4}$ b) $A = x_0 \omega^2$, $\delta = \frac{\pi}{4}$
 c) $A = x_0 \omega^2$, $\delta = -\frac{\pi}{4}$ d) $A = x_0 \omega^2$, $\delta = \frac{3\pi}{4}$

\Rightarrow (d) Displacement is
 $x = x_0 \cos(\omega t - \pi/4)$

$$\therefore v = \frac{dx}{dt} = -x_0 \sin(\omega t - \frac{\pi}{4}) \cdot \frac{d}{dt}(\omega t)$$

$$= -\omega x_0 \cdot \sin(\omega t - \pi/4)$$

$$\& a = \frac{dv}{dt} = -\omega^2 x_0 \cdot \cos(\omega t - \pi/4)$$

$$= \omega^2 x_0 \cdot \cos[\pi + (\omega t - \frac{\pi}{4})] \quad \left\{ \begin{array}{l} \cos(\pi + \theta) \\ = -\cos \theta \end{array} \right.$$

$$= \omega^2 x_0 \cdot \cos(\omega t + \frac{3\pi}{4}) \quad \left\{ \pi - \frac{\pi}{4} = \frac{3\pi}{4} \right.$$

Comparing this with $a = A \cos(\omega t + \delta)$, we get

$$A = \omega^2 x_0 \quad (\& \delta = \frac{3\pi}{4})$$

- (20) Two simple harmonic motion are represented by the following equations

$$y_1 = 10 \sin \frac{\pi}{4}(12t + 1)$$

$$\& y_2 = 5 (\sin 3\pi t + \sqrt{3} \cos \pi t).$$

The ratio of their amplitudes is 'm' & ratio of their time periods is 'n'. Then

- a) $m = \frac{3}{2}$ b) $m = 1$ c) $m = \frac{2}{3}$ d) $m = \frac{1}{2}$

⇒ (b) i) The two S.H.M. are given by -

$$y_1 = 10 \sin \frac{\pi}{2} (12t + \frac{\pi}{4})$$

$$= 10 \sin \left(\frac{12\pi t}{4} + \frac{\pi}{4} \right)$$

$$= 10 \sin \left(3\pi t + \frac{\pi}{4} \right) \quad \text{--- (1)}$$

∴ Amplitude of (1) = 10 units.

$$\& y_2 = 5 \sin 3\pi t + 5\sqrt{3} \cos(3\pi t) \quad \text{--- (2)}$$

$$= 5 \sin 3\pi t + 5\sqrt{3} \sin \left(\frac{\pi}{2} + 3\pi t \right)$$

$$\therefore \text{Amplitude} = \sqrt{5^2 + (5\sqrt{3})^2}$$

$$= \sqrt{25+75} = \sqrt{100} = 10$$

∴ Amplitude of (2) is = 10 units.

$$\therefore m = \frac{10}{10} = 1$$

ii) In each case, $T = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi} = \frac{2}{3}$ s

& Period of (1) = Period of (2) = $\frac{2}{3}$ s

$$\omega : n = \frac{2\pi}{T} = \frac{2\pi}{\frac{2}{3}} = 3\pi$$

② What is the nature of the graph between K.E. & P.E. of a particle performing a linear S.H.M.?

- a) a straight line passing through the origin
- b) a straight line parallel to the Ep axis.
- c) a straight line having intercepts on the Ep & Ek axes.
- d) a straight line parallel to Ep axis.

⇒ (c) $P.E. = \frac{1}{2} kx^2 \quad K.E. = \frac{1}{2} k(A^2 - x^2)$

mean

posn

($x=0$)

Extreme

posn

($x=A$)

$$\therefore P.E. = 0$$

$$K.E. = \frac{1}{2} k A^2$$

EP

$\therefore P.E. = \frac{1}{2} k A^2$

$\therefore P.E. = 0$

$\therefore P.E. =$

- (22) The equation of linear s.m.m. of a particle is given by $x = 0.5 \sin(\omega t + \frac{\pi}{3})$,
 the term $\frac{\pi}{3}$ represents
 a) the phase of s.m.m.
 b) the initial phase of s.m.m.
 c) angular frequency of s.m.m.
 d) period of s.m.m.

\Rightarrow (b) $x = 0.5 \sin(\omega t + \frac{\pi}{3})$

Comparing this with

$$x = A \sin(\omega t + \phi)$$

Phase angle ϕ represents
 of s.m.m. at epoch.

$\therefore \phi = \frac{\pi}{3}$ represents initial phase.

- (23) The displacement of a particle performing a s.m.m. is given by

$$x = 12 \cos(\omega t + \alpha) \text{ cm.}$$

If at time t , the displacement of the particle is 6 cm, then the phase of the particle at that instant is given as
 a) $\frac{\pi}{6}$ rad b) $\frac{\pi}{4}$ rad c) π rad d) $\frac{2\pi}{3}$ rad.

\Rightarrow (c) $x = 12 \cos(\omega t + \alpha)$ when $x = 6 \text{ cm}$
 as it represents initial phase.

$$\therefore 6 = 12 \cos(\omega t + \alpha)$$

$$\therefore \cos(\omega t + \alpha) = \frac{6}{12} = \frac{1}{2}$$

$$\therefore \cos(\omega t + \alpha) = \cos \frac{\pi}{3}$$

$$\therefore \text{Phase } (\omega t + \alpha) = \frac{\pi}{3}$$